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Ways to develop soft skills in pre-service mathematics and physics teachers when studying certain topics of olympiad mathematics

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Abstract. The development of soft skills in pre-service teachers helps to improve the quality of teaching, create a positive classroom environment and ensure an individualized approach to learning. The integration of soft skills, in particular flexibility and adaptability to change, helps teachers to respond effectively to new teaching methods, technological innovations and the requirements of the modern educational environment. The authors of this article aimed to investigate the possibilities of developing soft skills within the educational components of the training programme for pre-service mathematics and physics teachers. The methodological basis of the study was the analysis of scientific literature, synthesis, formalization of scientific sources, description, comparison, and theoretical generalization of own pedagogical experience of teaching the course "Olympiad Mathematics". The results of the study reveal some ways of developing soft skills of future masters of secondary education of Sumy State Pedagogical University named after A.S. Makarenko in the process of studying the course "Olympiad Mathematics". At the same time, among the various topics of the course, the topic "Proof of Inequalities" was singled out. The peculiarities of applying the following methods and techniques to the proof of inequalities in the context of development of soft skills are revealed: the method of strengthening (weakening) of inequality, the method of mathematical induction, the Malikov's method, the method of transforming the common term in sums and its evaluation, the method of evaluating the terms of the product and introducing an additional factor, the method of applying the properties of functions to the proof of inequalities, the method of using the properties of the definite integral of a certain function on a segment. It has been found that the most common methods of proving inequalities have wide possibilities in the context of development of soft skills. Each of the methods of proving inequalities discussed in the article is illustrated by relevant examples with detailed explanations. It is proved that this topic opens up wide opportunities for the acquisition and development of a number of soft skills. The research allows identifying specific ways to improve soft skills in pre-service mathematics and physics teachers, which are practically significant for further development of these professions when studying specific topics of the "Olympiad Mathematics" course: using group work formats during seminars, problem-based learning in mathematics, applying digital technologies in the study of Olympiad Mathematics, self-analysis and reflection, using mentoring support technology, public speaking during seminars with presentations of own solutions

Keywords: professional training of pre-service teachers; development of hard and soft skills; future masters of secondary education; study of Olympiad Mathematics; proving inequalities; techniques for proving inequalities

INTRODUCTION

The evolutionary processes caused by the transition of humanity from an industrial to an information (post-industrial) society, as well as various challenges faced by the world in recent years (russia's military aggression against Ukraine, the COVID-19 epidemic), which require the transition to the use of various online learning platforms, pose an acute problem for education to create modern educational technologies and search for alternative forms of education, develop an innovative strategy for teaching and upbringing, etc. This means a radical change in the goals and content of education, a transition from an educational model aimed at developing knowledge, skills, and abilities to the use of innovative educational models, which is possible only if the forms and methods of managing educational processes are updated. With the transition to a post-industrial society, the knowledge, skills, and competences that future professionals must acquire in higher education institutions (HEIs) have undergone a significant transformation. The ability to use digital technologies and relevant applications, collectively referred to as digital literacy, as well as foreign language skills, are moving from the category of important qualities of modern professionals to the category of instrumental skills and competences that are as necessary as language literacy.

In the scientific discourse, the term "soft skills" is often used, which is interpreted by Ukrainian scholars as "soft skills", "flexible skills", "success skills", "social skills", etc. Unlike "hard skills" (which include specialized knowledge, skills, and abilities required to perform specific professional tasks), which are easier to define and measure, "soft skills" are quite difficult to directly identify, test and demonstrate. S. Keng (2023) studied the relationship between academic performance (the degree of development of hard skills) and soft skills. He concluded that soft skills can reduce differences in academic performance due to differences in cognitive abilities. In particular, this is why stakeholders pay attention to soft skills when hiring specialists and the accreditation of HEI study programmes takes into account how the study programme ensures the acquisition of soft skills that meet the goals and learning outcomes of higher education students during their studies at the HEI (Borodina *et al.*, 2020; Rudeshko, 2020; Munir, 2021). According to N.A. Tarasenkova (2019), soft skills include a variety of knowledge, skills, experience, and attitudes that cannot be automated as a whole. These aspects can be manifested as intuitive, non-verbalised forms of knowledge and skills, and often require conscious and subconscious (sometimes even superconscious) support in different situations. In many cases, the use of soft skills involves the deployment of a whole system of actions that may include both conscious and subconscious aspects, and partial verbalization, at least for some of their fragments. The study by M. Danao & K. Main (2023) identified 11 representative soft skills that are not directly related to a specific industry as non-specialised, career-important supra-professional competencies responsible for job success and high performance. They include critical thinking, complex problem-solving, creativity, interpersonal skills, people management skills, judgement and decision-making, emotional intelligence, customer focus, negotiation skills, and the ability to learn new things quickly and easily.

Typical soft skills can be summarized as "4Cs" – collaboration, communication, creativity and critical thinking (Thornhill-Miller *et al.*, 2023). In the study by N. Foster & A. Schleicher (2022) considered a theoretical model of creative thinking implemented at the level of specific tasks, as well as possible ways to measure some aspects of creative thinking. H. Tseng *et al.* (2019) studied the impact of soft skills on the academic performance of business students in online learning.

The problem of soft skills development is of particular importance in the system of training pre-service teachers as the basis for the implementation of the professional standard of a teacher (Bezlyudna & Dudnyk, 2021), as in this case soft skills and hard skills play an equal role, as emphasized by (Agcami & Doganii, 2021). It is important to improve teachers' own professional skills through the development of skills required for direct communication with children within the school curriculum. Continuing this study, the authors of this article aimed to explore the possibilities and ways of developing soft skills of pre-service mathematics and physics teachers during their professional training.

MATERIALS AND METHODS

To achieve the goal, the following research methods are used: analysis of scientific literature, synthesis, formalization of scientific sources, description, comparison, theoretical generalization of own pedagogical experience of teaching the course "Olympiad Mathematics" on the example of studying the topic "Proof of Inequalities" within the Educational and Professional Programme Secondary Education (Mathematics. Computer Science) (2022), Educational and Professional Programme Secondary Education (Physics. Mathematics) (2020) of the second (master's) level of higher education of Sumy State Pedagogical University named after D.D. Zhukovskyi. Methods such as analysis of scientific literature and synthesis made it possible to determine the basis of the study, namely: the term "soft skills" was studied, the components of "soft skills" were studied, the factors influencing the formation of the overall complex of "soft skills" were identified, and the factors that directly affect the development of individual components of "soft skills" were separately identified. By means of formalizing scientific sources, a holistic view of the problem of developing soft skills of pre-service mathematics and physics teachers has been formed. In order to more fully cover the issue of developing soft skills of pre-service mathematics and physics teachers, the object of the study was divided into two parts. In the first part of the study, the authors investigated what should be understood by the term "soft skills", analysed the ways of forming and developing soft skills in general and which of them are relevant in the modern educational process of professional training of pre-service mathematics and physics teachers. In the second part of the study, the pedagogical experience of teaching the course "Olympiad Mathematics" at Sumy State Pedagogical University named after A.S. Makarenko was studied by comparing the Educational and Professional Programmes Secondary Education (Physics. Mathematics) of the second (master's) level of higher education. The article considers the possibilities of developing soft skills in pre-service mathematics and physics teachers on the example of studying one of the key topics of Olympiad Mathematics – the topic "Proof of Inequalities".

In the article, the authors describe the possibilities of introducing tasks for self-solving using online mathematical calculators, ChatGPT 3.5 artificial intelligence technology, and GeoGebra dynamic mathematics software. The data obtained is the basis for making appropriate changes to the educational process in the course "Olympiad Mathematics". When working with students, we followed the recommendations on the ethical aspect of conducting pedagogical research developed by reputable organizations, in particular The Code of Ethics of the American Educational Research Association (2011) and Ethical Guidelines for Educational Research (2018). At the next stage of the study, the method of theoretical generalization was used to develop recommendations on the use of forms and methods of teaching that would contribute to solving the problem of forming and developing soft skills pre-service of mathematics and physics teachers in the process of their professional training.

RESULTS AND DISCUSSION

One of the basic skills, the development of which is given great attention in the system of training pre-service mathematics and physics teachers, is the ability to prove mathematical statements. This skill includes, in particular: studying ready-made samples of mathematical proofs and the ability to reproduce them; conducting proofs by analogy; independent search for a method of proof; conducting a proof using the specified method; the ability to analyse different methods of proof, compare them and determine the advantages and disadvantages of each; the ability to choose a more rational method of proof among several methods; the ability to apply and synthesize specific mathematical facts and methods in the course of reasoning; the ability to use digital tools and tools for checking hypotheses; and the ability to use the tools to verify hypotheses. In addition, a pre-service mathematics teacher should not only be able to prove the theorems of the school mathematics course and mathematical statements provided for in the school curriculum, but also be able to "convey" these proofs to students, teach them to conduct similar reasoning on their own, and correctly point out students' mistakes. Thus, the ability to prove mathematical statements harmoniously combines hard skills and soft skills.

Author L. Lippman *et al.* (2015) identify five groups of soft skills:

- cognitive competences (innovative thinking, critical thinking, intellectual workload management, digital skills, self-education skills);
- social competences (interpersonal skills, leadership skills, social intelligence, responsibility);
- communication competences (communication skills and ethics);
- self-management skills (ability to control behaviour and manage emotions, ability to direct and focus attention, time management);
- positive self-esteem skills (emotional intelligence, empathy).

G. Duffy & T. Gallagher (2017) emphasize the importance of acquiring and developing a range of soft skills related to communication, including the ability to defend one's point of view, argue, and the ability to ask constructive questions in the course of collective activities. The development of the ability to prove mathematical statements in pre-service mathematics teachers takes place in all fundamental mathematics courses and in certain methodological disciplines (for example, in the courses "Methods of Teaching Mathematics", "Methods of Teaching Physics", "Elementary Mathematics", "Selected Issues of the School Physics Course", etc.) However, exceptional opportunities for the development of both this skill and related soft skills are provided by the course "Olympiad Mathematics", which is part of the programmes for the training of teachers of mathematics in Secondary Education (Mathematics. Computer Science) (2022) and physics Secondary Education (Physics. Mathematics) (2020) of the second (master's) level of higher education of Sumy State Pedagogical University named after A.S. Makarenko. This course is based on the hard skills of pre-service mathematics teachers mathematics teachers (knowledge of elementary mathematics, number theory, mathematical analysis, ability to apply them to solving typical problems), and on the other hand, it requires a certain level of soft skills (well-developed mathematical intuition, critical thinking, ability to apply knowledge in non-standard situations, ability to apply heuristic techniques, including the use of digital technologies, high level of communication skills, etc.) The ability to solve non-standard problems indicates a deep mastery of the mathematical apparatus, which in many cases is much more important than the so-called "pure knowledge" that can always be replenished.

Next, it is worth considering the prospects of developing soft skills in pre-service mathematics and physics teachers on the example of studying one of the basic topics of Olympiad Mathematics – "Proof of Inequalities". The tasks for proving inequalities are quite complex and have a high heuristic load. They allow for a deeper consolidation of a wide range of theoretical issues studied in the school mathematics course (inequality theory, equivalence of transformations, properties of functions, application of derivatives and integrals, geometric and vector inequalities and evaluations, etc.), contribute to the formation of critical thinking (in this case, the ability to critically analyse information, see logical connections and violations of logic in the proof of statements, the ability to indicate the grounds for errors and correct them, argue their opinions, the availability of justification for the statements, etc. It is for these reasons that tasks involving the proof of inequalities are popular among numerous mathematical competitions and contests.

In the proof of inequalities, *general methods of proving mathematical statements* are used (the method from the opposite, analytical and/or synthetic method, method of mathematical induction), *special methods of proving inequalities* (by definition of inequality, strengthening or weakening of inequality, reduction to obvious or well-known or classical inequalities, etc.), as well as artificial techniques based on *the properties of vectors, sequences, number series, certain properties of functions* (monotonicity, boundedness, convexity up or down), *the use of derivative and definite integral, various geometric estimates and characteristics*. The above list is rather arbitrary, and in practice it is often necessary to combine several methods simultaneously. Among the common approaches used in proving inequalities are the following: *transforming one or both parts of the inequality to a form convenient for its further proof; using the method of substitution* (for example, replacing the variables included in the correct inequality with some expressions in order to obtain the desired inequality); *considering a special case with further generalization of the idea of proof; breaking the inequality into several simpler ones and proving the latter; generalizing the inequality to be proved (for example, by the number of variables)* – in some cases, the study of a general statement requires less effort than a partial one. Special ideas and techniques for transforming inequalities used to prove them include: *adding the same expression to both parts of a correct inequality; adding or multiplying several correct inequalities of the same sign by parts; strengthening (respectively, weakening) inequalities*; and *using classical inequalities*. Let's consider the specifics of applying some of these methods and techniques to proving inequalities in the context of development of soft skills. One of the most popular methods of proving inequalities is the method of strengthening (weakening) the inequality.

1. Strengthening (weakening) inequality.

This method is used to one degree or another in proving many inequalities, and consists in the fact that to prove an inequality $a < b$, two inequalities are proved: $a < c$ and $c < b$. It is worth considering the application of this method on an example.

Example 1. Prove the inequality:

The left part of the inequality is to be strengthened as follows:

$$\underbrace{\sqrt{6+\sqrt{6+\ldots+\sqrt{6+\sqrt{6}}}}}_{2023}+\underbrace{\sqrt[3]{6+\sqrt[3]{6+\ldots+\sqrt[3]{6+\sqrt[3]{6}}}}}_{2024}<5. \qquad (1)$$

The next idea is to use the method of mathematical induction.

$$\underbrace{\sqrt{6+\sqrt{6+\ldots+\sqrt{6+\sqrt{6}}}}}_{2023}+\underbrace{\sqrt[3]{6+\sqrt[3]{6+\ldots+\sqrt[3]{6+\sqrt[3]{6}}}}}_{2024}<$$

$$<\underbrace{\sqrt{6+\sqrt{6+\ldots+\sqrt{6+\sqrt{9}}}}}_{2023}+\underbrace{\sqrt[3]{6+\sqrt[3]{6+\ldots+\sqrt[3]{6+\sqrt[3]{8}}}}}_{2024}<\sqrt{9}+\sqrt[3]{8}=3+2=5. \qquad (2)$$

2. Application of the method of mathematical induction.

The method of mathematical induction is used to prove mathematical statements that depend on a natural (rarely, an integer) parameter, and are true for all values of this parameter. This method is based on the principle of mathematical induction: *let some statement* T(n), *that depends on the natural n, be true for $n=1$ (or for $n=n_0$, $n_0 \in \mathbb{N}$), and from the assumption that it is true for $n=k$ follows its truth for natural $n=k+1$, then this statement is true for all natural n (with all natural $n \geq n_0$ respectively).*

Example 2. Prove that for arbitrary natural $n \geq 2$, inequality is fulfilled:

$$\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}<1-\frac{1}{n}. \quad (3)$$

Let's apply the method of mathematical induction and check the truth of the statement for $n = 2$. Since $\frac{1}{2^2}=\frac{1}{4}<1-\frac{1}{2}$, the inequality is correct.

Suppose that the inequality holds for $n=k$, i.e.:

$$\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{k^2}<1-\frac{1}{k}, \quad (4)$$

and it will be proved that the inequality is correct for $n=k+1$:

$$\underbrace{\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{k^2}}_{supposing\ that\ <1-\frac{1}{k}}+\frac{1}{(k+1)^2}<1-\frac{1}{k}+\frac{1}{(k+1)^2}=$$

$$=1+\frac{-(k+1)^2+k}{k(k+1)^2}==1+\frac{-k(k+1)-1}{k(k+1)^2}=$$

$$=1-\frac{1}{k+1}-\frac{1}{k(k+1)^2}<1-\frac{1}{k+1}, \quad (5)$$

i.e., the inequality is valid for $n=k+1$. Therefore, according to the principle of mathematical induction, this inequality holds for an arbitrary one natural $n \geq 2$. In the third step, when transforming the inequality, the assumptions of induction and amplification by discarding the negative multiplier were used. The method of mathematical induction is a fairly standard algorithm for proving mathematical statements, however, when performing an inductive transition in tasks for proving inequalities, the ability to "see" the desired result and discard the extra is largely used, and these are soft skills.

It is worth paying attention to one interesting modification of the method of mathematical induction – ***Malikov's method***, which greatly simplifies the verification of the statement at the inductive step. Its practical application to the proof of inequalities is based on the verification of the fulfilment of statements of one of the following schemes.

Scheme 1. *Let $n \in N_0$ and the conditions are fulfilled:*

1. $a_k > b_k$ for $k \geq 0$
2. $a_{n+1}-a_n > b_{n+1}-b_n$ at $n \geq k$

Then $a_n > b_n$ for every $n \geq k$.

Scheme 2. *Let $n \in N_0$ all terms of sequences a_n and b_n be positive and the conditions are fulfilled:*

1. $a_k > b_k$ for $k \geq 0$
2. $\frac{a_{n+1}}{a_n} > \frac{b_{n+1}}{b_n}$ at $n \geq k$

Then $a_n > b_n$ for every $n \geq k$.

It is worth applying Malikov's method to prove the inequality from **example 1**:

$$\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}<1-\frac{1}{n}, n \geq 2.$$

It is worth using scheme 1 and considering sequences with general terms $a_n=1-\frac{1}{n}$, $b_n=\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}$, $n \geq 2$. Since $a_2=1-\frac{1}{2}$, $b_2=\frac{1}{2^2}$, $a_2 > b_2$. It is worth considering the differences:

$$a_{n+1}-a_n=1-\frac{1}{n+1}-\left(1-\frac{1}{n}\right)=\frac{1}{n}-\frac{1}{n+1}=\frac{1}{n(n+1)}$$

$$b_{n+1}-b_n=\left(\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{(n+1)^2}\right)-$$

$$-\left(\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}\right)=\frac{1}{(n+1)^2}, \quad (6)$$

and $a_{n+1}-a_n=\frac{1}{n\cdot(n+1)}>\frac{1}{(n+1)^2}=b_{n+1}-b_n$, then by the Malikov's method $a_n > b_n$ for an arbitrary natural $n \geq 2$, i.e., $1-\frac{1}{n}>\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}$.

Thus, Malikov's method is simpler compared to the method of mathematical induction, although its application requires the ability to "see" the required sequences. It should be noted that the direct use of the method of mathematical induction (as well as Malikov's method) to prove the specified type of inequalities is not always possible, especially if one of the parts of the inequality is a number. It is worth considering the following task as an illustration.

Example 3. Prove that for arbitrary natural $n \geq 2$ inequality is fulfilled:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n\cdot(n-1)}<1. \quad (7)$$

In this case, when proving the inequality by the method of mathematical induction, at the stage of performing the inductive step, the following is obtained:

$$\underbrace{\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{k\cdot(k-1)}}_{supposing\ that\ <1}+\frac{1}{k\cdot(k+1)}<1+\frac{1}{k\cdot(k+1)}, \quad (8)$$

and further strengthening of the inequality, as in the previous example, is impossible. In such cases, it is advisable to use either more accurate estimates or other methods of proof. In this regard, we will consider the following method.

3. Conversion of the common term into sums and its evaluation.

This method involves transforming (evaluating, replacing, or the same transformation) the common term of the sum appearing in the inequality so that it can be calculated directly.

Example 4. Prove that for arbitrary natural $n \geq 2$ inequality is fulfilled:

$$\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}>\frac{1}{2}. \quad (9)$$

It is worth replacing each fraction of the amount on the left part of the inequality with a smaller $\frac{1}{k}>\frac{1}{2n}$, $k<2n$:

$$\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}>\underbrace{\frac{1}{2n}+\frac{1}{2n}+\ldots+\frac{1}{2n}}_{n}=n\cdot\frac{1}{2n}=\frac{1}{2}, \quad (10)$$

which had to be proved.

It is worth returning to **example 3** and writing the general term on the left part of the inequality in the form: $\frac{1}{k\cdot(k-1)}=\frac{1}{(k-1)}-\frac{1}{k}$. Next, each term of the sum is replaced with the specified difference, the following is obtained:

$$\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\cdots+\frac{1}{n\cdot(n-1)}=\left(1-\frac{1}{2}\right)+$$

$$+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{(n-1)}-\frac{1}{n}\right)=1-\frac{1}{n}<1, \qquad (11)$$

which had to be proved.

When proving the above inequality, it is possible to use one more non-standard technique in the context of this topic, related to the use of digital technologies. It is worth finding the amount on the left part of the inequality using an online calculator (Fig. 1).

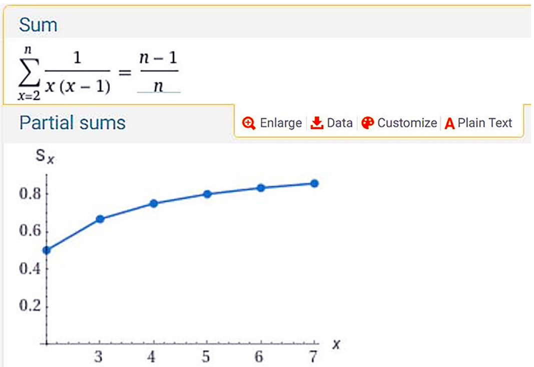

Figure 1. Using an online calculator to solve the inequality from example 2
Source: WolframAlpha (n.d)

Therefore, the sum of the expression on the left part of the inequality is equal to $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\cdots+\frac{1}{(n-1)\cdot n}=\frac{n-1}{n}$, and the inequality itself follows from the obvious estimate $\frac{n-1}{n}<1$ for all $n\geq2$. An obvious advantage of using an online calculator is the opportunity to see how to prove this inequality. At the same time, proving the identity $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\cdots+\frac{1}{(n-1)\cdot n}=\frac{n-1}{n}$, for example, by the same method of mathematical induction or replacing the k term of the sum by the difference $\frac{1}{k\cdot(k-1)}=\frac{1}{k-1}-\frac{1}{k}$, is a much simpler task than proving an inequality.

In this case, the ability to correctly and motivatedly apply digital technologies when proving inequalities contributes to the development of mathematical intuition and can be fully classified as soft skills. For this purpose, we offer students to analyse the solution of example 3 with the help of artificial intelligence technology ChatGPT 3.5 among tasks for independent solving. It should be noted that ChatGPT mainly uses the method of mathematical induction to solve problems for proving inequalities. However, it is example 3 that the system solves by replacing the term of the sum with a difference $\frac{1}{k\cdot(k-1)}=\frac{1}{k-1}-\frac{1}{k}$ (Fig. 2). However, the analysis of the solution, proposed by ChatGPT 3.5 artificial intelligence technology shows that the system incorrectly applies the decomposition and does not "see" terms with opposite signs, which should cancel each other out. This is where the mistake in the proof comes from, which the students must explain and correct on their own (Fig. 3).

"For any natural number $n\geq2$ prove the inequality:

$\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}<1-\frac{1}{n}$

Let's decompose the left part:

$\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}$

First, using the fact that $\frac{1}{k^2}<\frac{1}{k(k-1)}$ for any $k\geq2$.

It follows that $\frac{1}{k^2}>\frac{k}{k(k-1)}$ for all $k\geq2$.

Now we apply this inequality for every k from 2 to n:

$\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}<\frac{1}{2\cdot1}+\frac{1}{3\cdot2}+\ldots+\frac{1}{n(n-1)}$"

Figure 2. Solution of example 3 using artificial intelligence technology ChatGPT 3.5

"Many terms in the middle part cancel out:

$\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}<\left(\frac{1}{2}-\frac{1}{3}\right)+$
$+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{n-1}-\frac{1}{n}\right)+\frac{1}{n}$

Now, if we add $\frac{1}{n}$ to both sides of the inequality, we get:

$\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}+\frac{1}{n}<$
$<\left(\frac{1}{2}+\frac{1}{n}\right)+\left(\frac{1}{3}+\frac{1}{n}\right)+\ldots+\frac{1}{n}$

After simplification, we obtain:

$\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}+\frac{1}{n}<1-\frac{1}{n}$ ✓"

Figure 3. Mistake in solving task 3 using ChatGPT 3.5 artificial intelligence technology

The task of finding mistakes in inequality proofs has several useful aspects for students: developing critical thinking, enriching understanding of the topic, increasing responsibility, increasing the level of mastery of the material, which correlate with the task of developing the soft skills of pre-service mathematics and physics teachers by means of Olympiad mathematics. Such tasks stimulate students to carefully read and understand the material, as well as to identify possible flaws or inaccuracies in the logic of proofs, which contributes to the development of their critical thinking and analytical skills. By looking for mistakes in proofs, students deepen their understanding of the material, which can help them improve their understanding of the theory or concepts they are studying. The task of finding mistakes increases students' responsibility for their learning, as they learn to independently check and analyse the learning material to identify possible errors. However, the ability to detect mistakes in logic and reasoning is an important skill in many areas of both professional and everyday life, including scientific research, programming, and others. Also, identifying and correcting mistakes can improve the level of understanding of the material. In general, the task of finding mistakes in proofs is an effective tool for stimulating learning and developing students' cognitive skills.

We also pay attention to the fact that the inequalities from examples 2 and 3 make it possible to generate a whole series of new inequalities, in particular:

1) *Prove that for* $n \in \mathbb{N}, n \geq 2$: $\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}<1$.
2) *Prove that for* $n \in \mathbb{N}, n \geq 2$: $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{(n-1)\cdot n}<\frac{n}{n+1}$.
3) *Prove that for* $n \in \mathbb{N}, n \geq 2$: $\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{2023^2}<\frac{3}{4}$.
4) *Prove that for* $n \in \mathbb{N}, n \geq 2$: $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{2022\cdot 2023}<1$.
5) *Provethatfor* $n \in \mathbb{N}, n \geq 2$: $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{2022\cdot 2023}<\frac{2023}{2024}$.

The proof of inequalities 1)–2) can be carried out either by the methods specified above, or by using the results of examples 1 and 2 in combination to strengthen inequalities. For example, to prove the inequality, $\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}<1$ we evaluate the general term: $\frac{1}{k^2}<\frac{1}{(k-1)\cdot k}$:

$$\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}<\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{(n-1)\cdot n}. \quad (12)$$

Since the right part of the inequality is less than 1, the left side will also be less than 1.

To prove the inequalities 3)-5), it is necessary to first apply the transition to the general problem by introducing the variable n, and carry out the proof of the following inequalities:

3\`) $\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}<\frac{3}{4}$ for $n \in \mathbb{N}, n \geq 2$;
4\`) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n\cdot(n-1)}<1$ for $n \in \mathbb{N}, n \geq 2$;
5\`) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n\cdot(n-1)}<\frac{n-1}{n}$ for $n \in \mathbb{N}, n \geq 2$.

It is worth considering another heuristic technique, which uses the idea of strengthening inequalities.

4. Evaluation of terms of the product and introduction of an additional factor.

Quite often, when proving inequalities that contain products, the following idea can work: to prove an inequality $a<b$ for positive a and b, prove the inequality $ab<b^2$ (or $a^2<b^2$, or $a^2<ab$).

It is worth considering the application of this method on an example.

Example 5. Prove that for arbitrary $n \in \mathbb{N}$:

$$\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\ldots\cdot\frac{2n-1}{2n}\leq\frac{1}{\sqrt{2n}}. \quad (13)$$

Let's replace each term of the product with a larger fraction:

$$\frac{1}{2}<\frac{2}{3},\ \frac{3}{4}<\frac{4}{5},\ \frac{5}{6}<\frac{6}{7},\ldots,\frac{2n-1}{2n}<\frac{2n}{2n+1}, \quad (14)$$

and multiply the given inequalities:

$$\underbrace{\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\ldots\cdot\frac{2n-1}{2n}}_{A}<\underbrace{\frac{2}{3}\cdot\frac{4}{5}\cdot\frac{6}{7}\cdot\ldots\cdot\frac{2n}{2n+1}}_{B}. \quad (15)$$

We have $A<B$. It is worth multiplying both parts by $A>0$: $A^2<AB$, from where $A<\sqrt{AB}$ or:

$$\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\ldots\cdot\frac{2n-1}{2n}=A<\sqrt{AB}=$$
$$=\sqrt{\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\frac{5}{6}\cdot\ldots\cdot\frac{2n-1}{2n}\cdot\frac{2n}{2n+1}}=\sqrt{\frac{1}{2n+1}}\leq\frac{1}{\sqrt{2n}}, \quad (16)$$

which must be proved.

It is worth considering the application ***of properties of functions*** to proving inequalities. In this context, the properties of boundedness and monotonicity of functions, finding the largest and smallest values, using the derivative, constructing graphic images of the function, etc. are useful. As an illustration, consider the use of a graph of a function to estimate its values on a given interval.

Example 6. Prove that for $0<x<\frac{\pi}{2}$ the inequality is fulfilled:

$$\sin\sqrt{x}<\sqrt{\sin x}. \quad (17)$$

It is worth considering the function $y=\sin\sqrt{x}<\sqrt{\sin x}$ and examining its sign at the specified interval. To plot the graph of this function, we will use the dynamic mathematics software GeoGebra (Fig. 4).

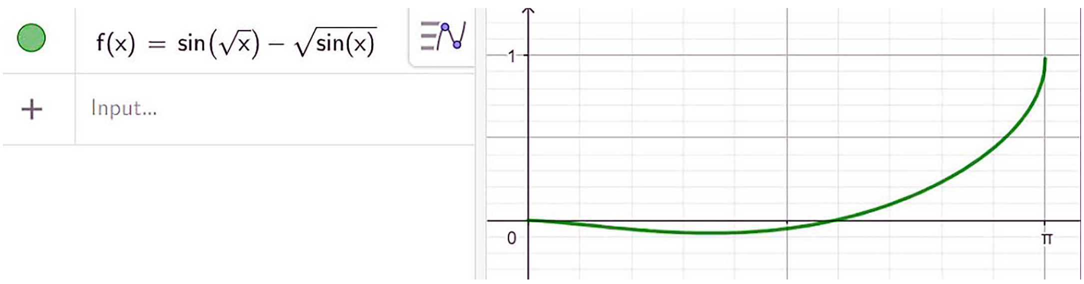

Figure 4. Application of the GeoGebra environment to solving example 5
Source: GeoGebra (n.d)

Since $0<x<\frac{\pi}{2}$ the specified function takes on negative values, when $\sin\sqrt{x}-\sqrt{\sin x}<0$, $\sin\sqrt{x}<\sqrt{\sin x}$.

It is worth noting that the given method allows "constructing" several obvious inequalities:

6) *Prove that in* $\sqrt{2}-\sqrt{\sin 2}$;
7) *Prove that* $(\sqrt{\cos 1}<\sqrt{\sin(\cos 1)}$;
8) *Prove that* $\sin\sqrt{\frac{1}{n^2}+2}>\sqrt{\sin\left(\frac{1}{n^2}+2\right)}$, $n \in \mathbb{N}$.

Of course, a strict justification for graphing a function $y=\sin\sqrt{x}<\sqrt{\sin x}$ should be done using the derivative, but using the capabilities of GeoGebra gives a hint as to exactly how to proceed in this case. It was presented another method of proving inequalities, which is based on estimates of integral sums and the use of properties of the definite integral of some function on the segment.

5. Use of integral properties for monotonic functions.

The use of this method is largely based on the monotonicity property of the integral, the mean value theorem, the value of the integral of monotonic and convex functions, etc. It is worth considering the application of these ideas on examples.

Example 7. Prove that for there is an inequality $e^{x} - 1 \geq x + \frac{x^2}{2}$.

To prove this inequality, it is worth applying *the monotonicity property of the integral* for continuous functions: *if the functions f(x) and g(x) are continuous and f(x) ≤ g(x), where x* ∈ [a;b]:

$$\int_a^x f(t)dt \leq \int_a^x g(t)dt. \quad (18)$$

At $x \geq 0$ we have an obvious inequality $e^x \geq 1$. Let's take $g(x) = e^x$, $f(x) = 1$. Then, according to the above property of monotonicity of the integral for functions $f(x)$and $g(x)$, we obtain$\int_0^x e^t dt \geq \int_0^x 1dt$, from where $e_x \geq x + 1$ for any $x \geq 0$.

Using the same method to the inequality $ex \geq x + 1$, we obtain:

$$\int_0^x e^t dt \geq \int_0^x (1+t)dt \text{ or } e^x - 1 \geq x + \frac{x^2}{2}. \quad (19)$$

Example 8. Prove that for any natural n:

$$\frac{1}{1^3} + \frac{1}{2^3} + \ldots + \frac{1}{n^3} < 2. \quad (20)$$

Let's write the left part of the inequality in the form:

$$\frac{1}{1^3} + \frac{1}{2^3} + \cdots + \frac{1}{n^3} = 1 + \frac{1}{2^3}(2-1) + \cdots$$
$$+ \frac{1}{n^3}(n-(n-1)) = 1 + \sum_{k=2}^{n} \frac{1}{k^3}(k-(k-1)). \quad (21)$$

It is worth entering the function $f(x) = \frac{1}{x^3}$ and dividing the segment $[1, n]$ points $x_k = k$, $k = 1,2,\ldots,n$ on n of equal parts of length 1. Then the expression $\sum_{k=2}^{n} \frac{1}{k^3}(k-(k-1))$ is equal to the sum of the areas of rectangles built on segments $[k - 1;\ k]$ as on foundations with elevations $f(x_k) = f(k) = \frac{1}{k^3}$.

As it is known, for a continuous, positive and decreasing function $f(x)$ *on the interval* [a, b] there is an inequality (which follows from the method of rectangles):

$$\sum_{k=2}^{n} f(x_k)(x_k - x_{k-1}) < \int_a^b f(x)dx <$$
$$< \sum_{k=2}^{n} f(x_{k-1})(x_k - x_{k-1}). \quad (22)$$

Since the function $y = \frac{1}{x^3}$ at $x > 0$ is decreasing, continuous and positive:

$$1 + \sum_{k=2}^{n} \frac{1}{k^3}(k-(k-1)) < 1 + \int_1^n \frac{dx}{x^3} =$$
$$= 1 - \frac{1}{2x^2}\Big|_1^n = \frac{3}{2} - \frac{1}{2n^2} < 2, \quad (23)$$

which had to be proved.

The proof of the last inequality can be carried out in another way, using the evaluation of the common term of the left part, followed by the representation of the resulting fraction in the form of a difference (example 2):

$$\frac{1}{k^3} < \frac{1}{k^2} < \frac{1}{k\cdot(k-1)} = \frac{1}{k-1} - \frac{1}{k},\ k = 2,3,\ldots, \quad (24)$$

Then:

$$\frac{1}{1^3} + \frac{1}{2^3} + \cdots + \frac{1}{n^3} < \frac{1}{1^2} + \frac{1}{2^2} + \cdots + \frac{1}{k^2} + \cdots + \frac{1}{n^2} <$$
$$< 1 + \frac{1}{1\cdot 2} + \cdots + \frac{1}{k\cdot(k-1)} + \cdots + \frac{1}{n\cdot(n-1)} =$$
$$= 1 + \left(1 - \frac{1}{2}\right) + \cdots + \left(\frac{1}{k-1} - \frac{1}{k}\right) + \cdots$$
$$+ \left(\frac{1}{(n-1)} - \frac{1}{n}\right) = 2 - \frac{1}{n} \leq 2. \quad (25)$$

Therefore, $\frac{1}{1^3} + \frac{1}{2^3} + \cdots + \frac{1}{n^3} < 2$ for all natural n.

The use of the properties of the integral to prove inequalities provides a wide range of possibilities for creating new inequalities. For this purpose, both the above methods and various estimates of areas and volumes, which are direct applications of the definite integral, are used. As the authors' own experience shows, the problem of developing soft skills of pre-service mathematics and physics teachers can be effectively solved by studying the course of Olympiad Mathematics. In their previous studies (Lukashova *et al.*, 2022; Lukashova *et al.*, 2023), the authors also reveal the possibilities of developing soft skills within the educational components of programmes for the training of pre-service mathematics and physics teachers, which include heuristic workload. This study is consistent with the research by E.O. Cardoso-Espinosa *et al.* (2021) on the development of mathematical software skills at the higher education level through project-based learning in times of COVID-19. In particular, the expediency of group work during seminars is confirmed: it is advisable to organize group work of students on solving Olympiad tasks, as joint work contributes to the development of communication skills, the ability to cooperate and resolve conflicts. The group form of work allows including elements of personal development training in the course of Olympiad mathematics, which will help pre-service mathematics and physics teachers to understand and develop their soft skills, such as leadership, tolerance, and teamwork. The study confirms the experience of active learning of business students by D. Gil-Domenech & J. Berbegal-Mirabent (2020) and ideas on how to make learning mathematics meaningful. In particular, the authors found that problem-based learning of mathematics is appropriate, as the use of Olympiad-level problems often requires creativity, critical thinking and complex problem-solving, which contributes to the development of soft skills in pre-service teachers. As the results show, the study fully supports the work of C.M.A. Pinto *et al.* (2022) on ways to develop soft skills in students, in particular through the use of digital technologies: studying a course in Olympiad mathematics allows integrating the use of modern digital technologies into teaching through the use of video lectures on the study of theoretical material, virtual graphing tools, interactive platforms, which can make learning more attractive and contribute to the development of technological and communication skills.

This study is consistent with the research by R.Y. Stytsyuk *et al.* (2022), as mathematics learning has a high potential for development of soft skills. In particular, it is advisable to encourage pre-service mathematics and physics teachers to conduct self-analysis and reflection on their learning and development of soft skills. This will also contribute to the development of self-improvement and self-management. The use of Olympiad mathematics as a basis for soft skills development helps to create a comprehensive approach to learning that contributes to the formation of not only mathematical knowledge but also key competences for pre-service teachers. The study confirms the findings of P. Zimmermann *et al.* (2018) on the use of project-based learning and STEM education technologies for the development of soft skills. Indeed, studying a course in Olympiad Mathematics allows the use of mentoring technology, where students with a higher level of learning capabilities or a mathematics teacher can help students develop both professional and personal skills, and such forms of organizing students' learning activities as public speaking during seminars to present their solutions or teaching theoretical material for a task encourage them to overcome internal barriers to public speaking, which can help them to develop soft skills. In their previous studies (Lukashova *et al.*, 2022; Lukashova *et al.*, 2023), the authors studied the possibilities of developing soft skills within the educational components of the EPP for pre-service mathematics and physics teachers, including those that involve a significant heuristic component. This study confirmed that the educational components of these programmes (in particular, Algebra and Number Theory and Elementary Mathematics) contribute to the formation and development of both hard skills (thorough knowledge of a number of sections of elementary mathematics, higher algebra and number theory, the ability to apply them to solving basic problems, mastery of methods of proving inequalities, etc:

- ingenuity (ability to apply knowledge in atypical situations; ability to create ideas for proof, ability to apply heuristic techniques and methods, in particular, with the use of digital technologies);
- critical thinking (the ability to critically perceive and analyse information, to see logical connections in the proof of statements and their violations, the ability to rationally choose a particular method of proving an inequality, the ability to "see" the idea that gives the key to the solution; the ability to ask appropriate questions during the joint proof of inequalities in practical and seminar classes, the ability to analyse, justify and evaluate different approaches and methods of proof, the ability to identify errors and correct them, the presence of doubts);
- communication (ability to prove one's position, justify why a particular method of proof was chosen, be able to explain why other methods cannot be used to solve a given inequality);
- time management skills in determining deadlines for individual and homework assignments, etc., the ability to use their time rationally;
- the ability to learn independently (a significant part of the hours is allocated for self-study).

The results of the study show the expediency of group work, the use of group presentations to develop cooperation and teamwork skills of pre-service teachers. This approach is in line with current trends in education and supports the introduction of digital technologies to ensure effective learning.

CONCLUSIONS

The study notes that over the past 5 years, stakeholders in the accreditation of educational programmes of higher education institutions have been increasingly paying attention to such qualities of pre-service teachers as soft skills – abilities and skills that allow them to perform work effectively, creatively and efficiently. These are the qualities a pre-service teacher should have, as this is a profession in which soft skills and hard skills are equally important. In this context, mastering fundamental mathematical disciplines in the professional training of pre-service teachers should include the development of a balance between hard and soft skills.

The study examines the interpretation of the term "soft skills", analyses the components of soft skills, identifies the factors that influence the formation of the overall soft skills complex, and separately identifies the factors that directly affect the development of individual components of soft skills. The results of the study demonstrate different ways to solve the problem of developing soft skills of pre-service mathematics and physics teachers. For example, when studying the topic "Proof of Inequalities", the development of soft skills is greatly facilitated by organizing students' work in groups (both in seminars and at home), using problem-based learning in mathematics, using digital technologies both for modelling the problem and for analysing the finished solution generated by a chatbot with artificial intelligence, using mentoring technologies, public speaking and presenting their own solutions, self-analysis, and reflection. At the same time, the study highlights the peculiarities of applying the following methods and techniques to the proof of inequalities in the context of development of soft skills: the method of strengthening (weakening) inequality, the method of mathematical induction, the Malikov's method, the method of converting the common term in sums and its evaluation, the method of evaluating the terms of the product and introducing an additional factor, the method of applying the properties of functions to the proof of inequalities, the method of using the properties of the definite integral of a certain function on a segment.

Through theoretical generalization of their own pedagogical experience, the authors have determined that the ability to correctly and motivatedly apply digital technologies in proving inequalities contributes to the development of mathematical intuition and can be fully attributed to soft skills. The ways of developing soft skills of pre-service mathematics and physics teachers of in the process of studying the topic "Proof of Inequalities" are illustrated by the use of such digital tools as online mathematical calculators, artificial intelligence technologies ChatGPT 3.5, and dynamic mathematics software GeoGebra. The authors see the prospects for further research in studying the peculiarities of development of soft skills of mathematics and physics teachers during their in-service training.

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CONFLICT OF INTEREST

There is none.

REFERENCES

[1] Agcami, R., & Doganii, A. (2021). A study on the soft skills of pre-service teachers. *International Journal of Progressive Education*, 17(4), 35-48. doi: 10.29329/ijpe.2021.366.3.
[2] Bezlyudna, N., & Dudnyk, N. (2021). Formation of soft skills in future teachers as a condition for the realization of the teacher's professional standard. *Psychological and Pedagogical Problems of the Modern School*, 2(6), 137-143. doi: 10.31499/2706-6258.2(6).2021.248137.
[3] Borodina, N.A., Cheberiachko, S.I., & Shevchuk, N.A. (2020). Formation of soft skills and principles of academic integrity in students of higher education when teaching the discipline of the specialty "Civil Security". *News of the Donetsk Mining Institute*, 2(47), 206-214. doi: 10.31474/1999-981x-2020-2-206-214.
[4] Cardoso-Espinosa, E.O., Cortes-Ruiz, J.A., & Zepeda-Hurtado, E. (2021). The development of mathematics and soft skills at the graduate level through project-based learning in times of COVID-19. *TEM Journal*, 10(4), 1638-1644. doi: 10.18421/TEM104-20.
[5] Danao, M., & Main, K. (2023). 11 essential soft skills in 2023 (With examples). *Forbes Advisor*. Retrieved from https://www.forbes.com/advisor/in/business/soft-skills-examples/.
[6] Duffy, G., & Gallagher, T. (2017). Shared education in contested spaces: How collaborative networks improve communities and schools. *Journal of Educational Change*, 18, 107-134. doi: 10.1007/s10833-016-9279-3.
[7] Educational and professional program Secondary education (Mathematics. Informatics) of the second (master's) level of higher education of Sumy State Pedagogical University named after A.S. Makarenko. (2022). Retrieved from https://sspu.edu.ua/osvitni-prohramy-2022-rik.
[8] Educational and professional program Secondary education (Physics. Mathematics) of the second (master's) level of higher education of Sumy State Pedagogical University named after A.S. Makarenko. (2020). Retrieved from https://sspu.edu.ua/osvitni-prohramy-2022-rik.
[9] Ethical Guidelines for Educational Research. (2018). Retrieved from https://www.bera.ac.uk/resources/all-publications/resources-for-researchers.
[10] Foster, N. & Schleicher, A. (2022) Assessing creative skills. *Creative Education*, 13(1), 1-29. doi: 10.4236/ce.2022.131001.
[11] Gil-Domenech, D., & Berbegal-Mirabent, J. (2020). Making the learning of mathematics meaningful: An active learning experience for business students. *Innovations in Education and Teaching International*, 57(4), 403-412. doi: 10.1080/14703297.2020.1711797.
[12] Keng, S. (2023). The effect of soft skills on academic outcomes. *The B.E. Journal of Economic Analysis & Policy*. doi: 10.1515/bejeap-2022-0342.
[13] Lippman, L.H., Ryberg, R., Carney, K., & Moore, A. (2015). *Workforce connections: Key "soft skills" that foster youth. Workforce success: Toward a consensus across fields*. Washington, DC: United States Agency for International Development.
[14] Lukashova, T., Drushlyak, M., & Khvorostina, Yu. (2022). Development of pre-service mathematics teachers' soft skills studying the topic "Diophantine Equations". *Physical and Mathematical Education*, 36(4), 57-63. doi: 10.31110/2413-1571-2022-036-4-008.
[15] Lukashova, T., Drushlyak, M., & Khvorostina, Yu. (2023). Knowledge foundation in the system of professional training of pre-service mathematics teachers in studying the mathematical induction method. *Physical and Mathematical Education*, 38(3), 29-35. doi: 10.31110/2413-1571-2023-038-3-004.
[16] Munir, F. (2021). Do the engineering education institutions provide soft skills education? Views of South African engineering professionals. *South African Journal of Higher Education*, 35(4), 162-179. doi: 10.20853/35-4-4264.
[17] Pinto, C.M.A., Mendonca, J. & Nicola, S. (2022). Drive-math project: Case study from the Polytechnic of Porto, PT. *Open Education Studies*, 4(1), 1-20. doi: 10.1515/edu-2022-0001.
[18] Rudeshko, E.V. (2020). Learning technologies and innovative methods of teaching foreign languages. In *Soft Skills as integral aspects of students' competitiveness in the XXI century* (pp. 70-72). Kyiv: Kyiv National University of Trade and Economics.

[19] Stytsyuk, R.Y., Lustina, T.N., Sekerin, V.D., Martynova, M., Chernaysky, M.Y., & Terekhova, N.V. (2022). Impact of STEM education on soft skill development in it students through educational scrum projects. *Revista Conrado*, 18(84), 183-192.
[20] Tarasenkova, N.A. (2019). The formation of "soft skills" in teaching mathematics: Before posing a problem. In *V All-Ukrainian scientific and practical conference "Personally oriented teaching of mathematics: Present and prospects"* (pp. 22-23). Poltava: Astraia.
[21] The Code of Ethics of the American Educational Research Association. (2011). Retrieved from https://www.aera.net/About-AERA/AERA-Rules-Policies/Professional-Ethics.
[22] Thornhill-Miller, B., *et al.* (2023). Creativity, critical thinking, communication, and collaboration: Assessment, certification, and promotion of 21st century skills for the future of work and education. *Journal of Intelligence*, 11(3), article number 54. doi: 10.3390/jintelligence11030054.
[23] Tseng, H., Yi, X. & Yeh, H.T. (2019). Learningrelated soft skills among online business students in higher education: Grade level and managerial role differences in self-regulation, motivation, and social skill. *Computers in Human Behavior,* 95, 179-186. doi: 10.1016/j.chb.2018.11.035.
[24] WolframAlpha. (n.d). Retrieved from https://www.wolframalpha.com/.
[25] Zimmermann, P., Stallings, L., Pierce, R., & Largent, D. (2018). Classroom interaction redefined: Multidisciplinary perspectives on moving beyond traditional classroom spaces to promote student engagement. *Journal of Learning Spaces*, 7(1), 45-61.

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Шляхи розвитку soft skills у майбутніх учителів математики та фізики при вивченні окремих тем олімпіадної математики

Анотація. Розвиток soft skills у майбутніх вчителів допомагає підвищити якість навчання, створити позитивне середовище в класі та забезпечити індивідуалізований підхід до навчання. Інтеграція soft skills, зокрема гнучкість та здатність пристосування до змін, допомагає вчителю ефективно реагувати на нові методи навчання, технологічні інновації та вимоги сучасного освітнього середовища. Авторами даної статті була поставлена мета дослідити можливості розвитку soft skills у межах освітніх компонент програми підготовки майбутніх учителів математики та фізики. Методологічну основу дослідження склали аналіз наукової літератури, синтез, формалізація наукових джерел, опис, зіставлення, теоретичне узагальнення власного педагогічного досвіду викладання курсу «Олімпіадна математика». Результати дослідження розкривають деякі способи розвитку soft skills майбутніх магістрів середньої освіти Сумського державного педагогічного університету імені А.С. Макаренка у процесі вивчення курсу «Олімпіадна математика». При цьому серед різних тем курсу було виокремлено тему «Доведення нерівностей». Розкрито особливості застосування таких методів та прийомів до доведення нерівностей в контексті розвитку soft skills: метод підсилення (послаблення) нерівності, метод математичної індукції, метод Малікова, метод перетворення загального члена в сумах та його оцінка, метод оцінки членів добутку та введення додаткового множника, метод застосування властивостей функцій до доведення нерівностей, метод використанням властивостей визначеного інтеграла деякої функції на відрізку. З'ясовано, що найбільш поширені прийоми доведення нерівностей мають широкі можливості в контексті розвитку soft skills. Кожен з розглянутих у статті методів доведення нерівностей проілюстровано відповідними прикладами з детальними поясненнями. Доведено, що дана тема відкриває широкі можливості для набуття та розвитку цілої низки soft skills. Дослідження дозволяє визначити конкретні способи покращення soft skills у майбутніх вчителів математики та фізики, що мають практичне значення для подальшого розвитку цих професій при вивченні окремих тем курсу «Олімпіадна математика»: використання групових форм роботи під час проведення семінарських занять, проблемне вивчення математики, застосування цифрових технологій під час вивчення олімпіадної математики, самоаналіз та рефлексія, використання технології менторської підтримки, публічні виступи під час проведення семінарських занять з представленням власних розв'язків

Ключові слова: професійна підготовка майбутніх учителів; розвиток твердих та м'яких навичок; майбутні магістри середньої освіти; вивчення олімпіадної математики; доведення нерівностей; прийоми доведення нерівностей